**MATH 7B - TEST 2 v2**
**Fall 2024**
**(Unit 4)**

**100 POINTS**

**NAME:** Solutions

**Show your work on this paper. EXACT answers are expected unless otherwise specified. No Graphing Calculators. No scratch paper**

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$

$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

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Fill in the blanks. (2 points each)

(1) Express $\sin 4\theta \sin 3\theta$ as a sum $\frac{1}{2}(\cos\theta - \cos 7\theta)$

(2) True or false: $\sin 26^\circ = 2\sin 13^\circ =$ False

(3) Simplify the expression $\tan x \csc x$ $\sec x$

$\frac{\sin x}{\cos x}\cdot\frac{1}{\sin x} = \frac{1}{\cos x}$

(4) The power reducing formula for sine is $\sin^2(\theta) = \frac{1-\cos 2\theta}{2}$

(5) Express $\cos 6\theta + \cos 4\theta$ as a product $2\cos 5\theta \cos\theta$

(6) True or False: $\sin(a+b) = \sin a + \sin b$ False

(7) $\sin 15^\circ \cos 15^\circ =$ $\frac{1}{2}\sin 30^\circ = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ (exact, simplify)

$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

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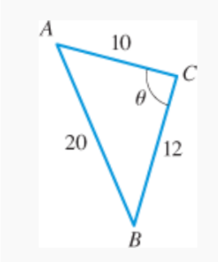
(8) Find θ, exactly. Then give the approximate value to 3 decimal places: (4 points)




$20^2 = 10^2 + 12^2 - 2(10)(12)\cos\theta$

$400 = \cancel{244 - 240}\cos\theta$ ← you cannot combine the 244 and the 240 If you we like saying 244-240x = 4x

$156 = -240\cos\theta$

$\cos\theta = \frac{-156}{240} = \frac{-13}{20}$

$\boxed{\theta = \cos^{-1}\left(\frac{-13}{20}\right) \approx 130.5^\circ}$

(9) Simplify: $\frac{\sec^2\theta - 1}{\sec^2\theta} = \frac{\sec^2\theta}{\sec^2\theta} - \frac{1}{\sec^2\theta}$ (6 points)

$= 1 - \cos^2\theta = \boxed{\sin^2\theta}$

(10) Using identities, find the exact, simplified value of the following: (3 points each)
(You must show work, for credit. Calculators should not be used on this problem)

(a) $\cos\left(\frac{5\pi}{12}\right)$ __________

$= \cos\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

(b) $\sin 67.5^\circ$ ________

$\sin 67.5 = \sin\left(\frac{135^\circ}{2}\right)$

$= \pm\sqrt{\frac{1 + \cos 135^\circ}{2}}$

$= \pm\sqrt{\frac{1 + \sqrt{2}/2}{2}}$

$= \boxed{\frac{\sqrt{2 + \sqrt{2}}}{2}}$

(11) Prove the following identity. Presentation should be very clear. (6 points)

$$\frac{\sin x + \cos x}{\sec x + \csc x} = \frac{1}{2}\sin 2x$$

$$\frac{\sin x + \cos x}{\sec x + \csc x} = \frac{\sin x + \cos x}{\frac{1}{\cos x} + \frac{1}{\sin x}} \quad \frac{\cos x \sin x}{\cos x \sin x}$$

$$= \frac{\sin x(\cos x \sin x) + \cos x(\cos x \sin x)}{\sin x + \cos x}$$

$$= \frac{\cos x \sin^2 x + \cos^2 x \sin x}{\sin x + \cos x}$$

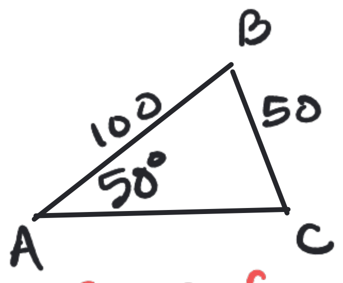
$$= \frac{\cos x \sin x(\sin x + \cos x)}{\sin x + \cos x} = \cos x \sin x$$

$$= \frac{1}{2}\sin 2x$$

So $\frac{\sin x + \cos x}{\sec x + \csc x} = \frac{1}{2}\sin 2x$

(12). Find, to the nearest tenth, the value of the remaining parts for all possible triangles satisfying the given conditions. $\angle A = 50^\circ$, $a = 50$ $c = 100$

(6 points)



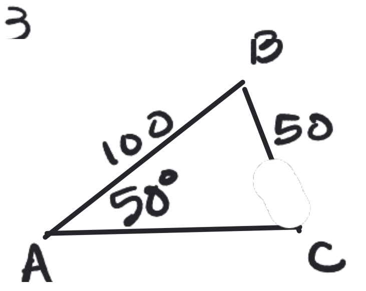

$$\frac{\sin 50^\circ}{50} = \frac{\sin C}{100}$$

$\sin C = 2\sin 50^\circ \approx 1.53$

no solution

$\Rightarrow$ $\boxed{\text{no such } \triangle}$

some of you went from $\sin C = 1.53$ to $C = 1.53$




(13)

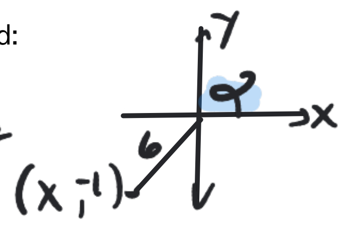
Given $\sin\alpha = -1/6$, $\alpha$ in the third quadrant, and $\tan\theta = 3/4$, $\pi < \theta < \dfrac{3\pi}{2}$

Find: (8 points)

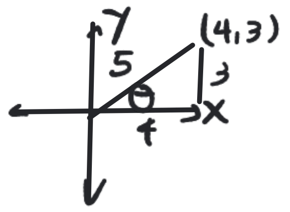
$x^2 + (-1)^2 = 6^2$

$x^2 = 35$

$x = \sqrt{35}$




$(-\sqrt{35}, -1)$




a) $\cos(\alpha - \theta) = \cos\alpha\cos\theta + \sin\alpha\sin\theta$

$= \frac{-\sqrt{35}}{6}\cdot\frac{4}{5} + \frac{-1}{6}\cdot\frac{3}{5} = \boxed{\frac{-4\sqrt{35}-3}{30}}$

b) $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\cdot\frac{3}{4}}{1-9/16} = \dfrac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2}\cdot\frac{16}{7} = \boxed{\frac{24}{7}}$

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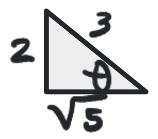
(14) Simplify exactly: $\sin\left(2\underbrace{\sin^{-1}(2/3)}_{\theta}\right) = \sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{2}{3}\cdot\frac{\sqrt{5}}{3}$ (4 points)

$\boxed{\frac{4\sqrt{5}}{9}}$

$\theta = \sin^{-1}\frac{2}{3}$

$\left\{\begin{array}{l}\sin\theta = \frac{2}{3} \\ \text{and} \\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\end{array}\right.$



SOLVE the following equations: $0 \le x < 2\pi$ (8 points each)

Dividing by cos x will lose solutions

(15) $\sin 2x = \sqrt{3}\cos x$

$2\sin x\cos x = \sqrt{3}\cos x$

$2\sin x\cos x - \sqrt{3}\cos x = 0$

$\cos x\,(2\sin x - \sqrt{3}) = 0$

$\cos x = 0 \qquad \sin x = \frac{\sqrt{3}}{2}$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}}$$

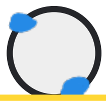
(16) $\tan^2 x + 6\tan x = -5$

$\tan^2 x + 6\tan x + 5 = 0$

$(\tan x + 5)(\tan x + 1) = 0$

$\tan x = -5 \qquad \tan x = -1$

ref: $\tan^{-1} 5$

$$\boxed{x = \pi - \tan^{-1}5,\ 2\pi - \tan^{-1}5,\ \frac{3\pi}{4}, \frac{7\pi}{4}}$$

Some said tan x = −5 has no solutions. That would be true for cos x = −5 or sin x = −5 but tangent's range is all real #.

(17) $\dfrac{1-\sec x}{1+\sec x} = -\dfrac{1}{3}$

$1 - \sec x = -\frac{1}{3}(1 + \sec x)$

$1 - \sec x = -\frac{1}{3} + -\frac{1}{3}\sec x$

$\frac{4}{3} = \frac{2}{3}\sec x$

$2 = \sec x$

$\cos x = \frac{1}{2}$

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$

(18) $\cos^2 x - 3\sin^2 x + 1 = 0$

$\cos^2 x - 3(1 - \cos^2 x) + 1 = 0$

$\cos^2 x - 3 + 3\cos^2 x + 1 = 0$

$4\cos^2 x - 2 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}$

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

OR

$\cos^2 x - 3\sin^2 x + 1 = 0$

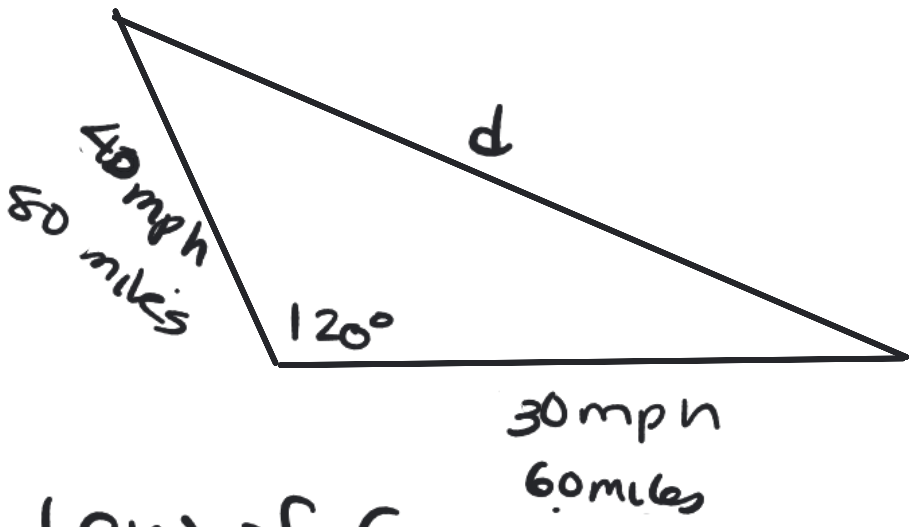
$1 - \sin^2 x - 3\sin^2 x + 1 = 0$

$2 - 4\sin^2 x = 0$

$\sin^2 x = 1/2$

$\sin x = \pm\frac{\sqrt{2}}{2} \ldots$

(19). Two cars leave a city at the same time and travel along straight highways that differ in direction by 128°. If their speeds are 60mph and 50mph respectively, how far apart are the cars at the end of 2 hours? (Exact and approximate.)
(*sketch a picture and label any variables used).* *(7 points)*




Law of Cosines

$$d^2 = 80^2 + 60^2 - 2 \cdot 80 \cdot 60 \cos 120°$$

$$d^2 = 6400 + 3600 - 9600\left(-\frac{1}{2}\right)$$

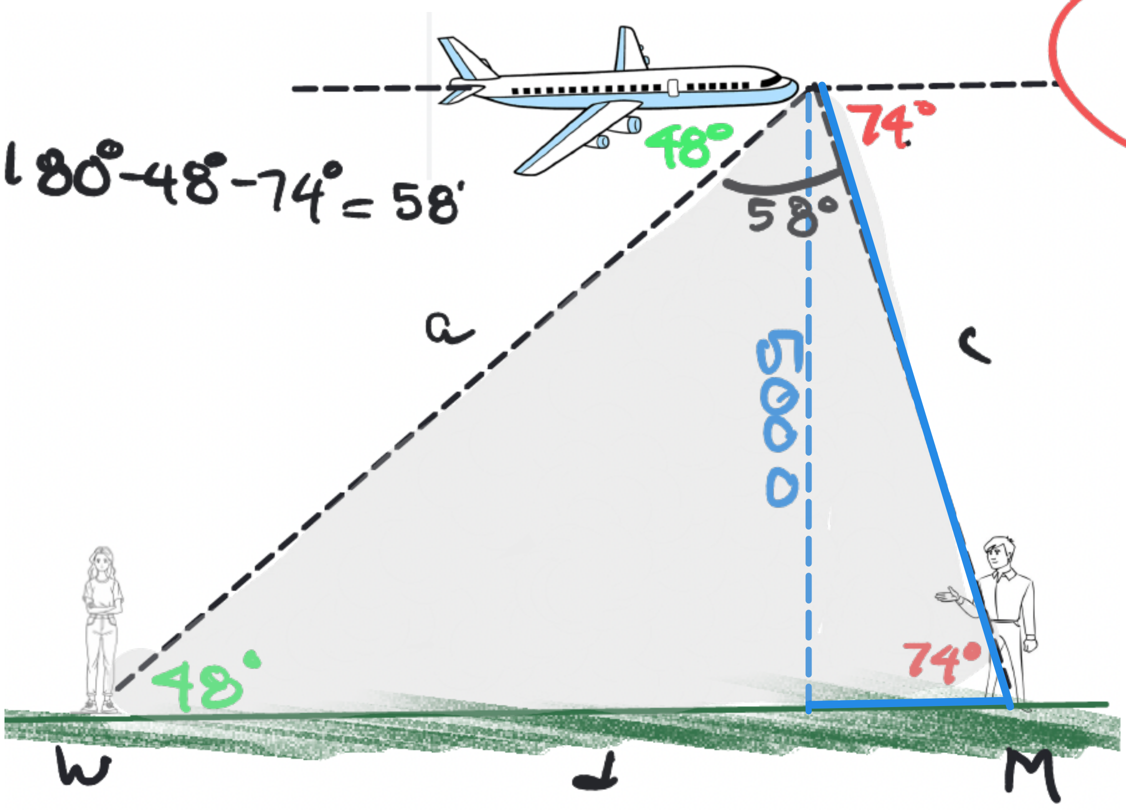
$$d^2 = 10000 + 4800$$

$$d^2 = 14800$$

$$\boxed{d = \sqrt{14800} = 121.7 \text{ miles}}$$

(20). An airplane is flying at an altitude of 5000ft over a field where a man and a woman are standing some distance apart. The pilot determines that the angle depression to the man is $74^\circ$ and the angle of depression to the woman is $48^\circ$. What is the distance between the man and the woman? *(label any variables used.)* (7 points)

Many ways you can do this




① Using law of Sines

Using the big △,

$$\frac{d}{\sin 58^\circ} = \frac{a}{\sin 74^\circ} = \frac{c}{\sin 48^\circ}$$

We need more information.

Consider the right triangle on the right

$$\sin 74^\circ = \frac{5000}{c}$$

$$c \sin 74^\circ = 5000$$

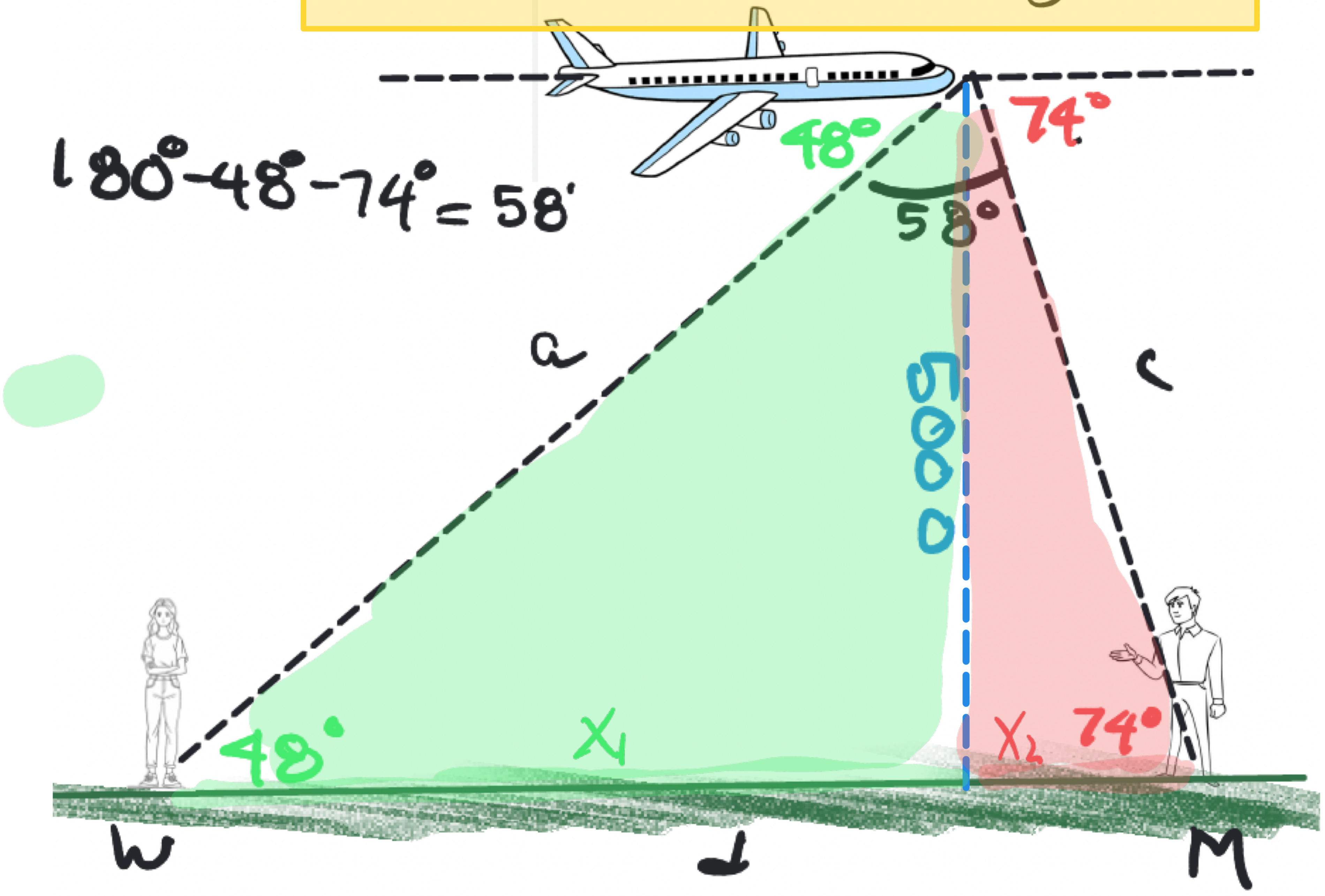
$$c = \frac{5000}{\sin 74^\circ}$$

So $$\frac{d}{\sin 58^\circ} = \frac{c}{\sin 48^\circ}$$

$$\Rightarrow d = \frac{c}{\sin 48^\circ} \sin 58^\circ = \frac{\frac{5000}{\sin 74^\circ}}{\sin 48^\circ} \sin 58^\circ = \frac{5000 \sin 58^\circ}{\sin 74^\circ \sin 48^\circ} \text{ (exact)}$$

$$\boxed{d \approx 5935.747 \text{ ft}}$$

# ② Using Right Triangles




$\tan 48° = \frac{5000}{x_1}$

$x_1 = \frac{5000}{\tan 48°}$

$\tan 74° = \frac{5000}{x_2}$

$x_2 = \frac{5000}{\tan 74°}$

$$d = x_1 + x_2 = \frac{5000}{\tan 48°} + \frac{5000}{\tan 74°} \approx 5935.747 \text{ ft}$$

Calculator Usage : We still need to work on calculator usage, storage and efficency. You should never need to write down intermediate computations, then enter them back in.